

THE SINGLE  $W$  PRODUCTION CASE

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INFN, Sezione di Torino, Italy<sup>†</sup>**Abstract**

The process  $e^+e^- \rightarrow e^-\bar{\nu}_e f_1 \bar{f}_2$ , belonging to the so-called CC20 family, has been extensively analyzed in the literature. It is a sensitive probe of anomalous electromagnetic couplings of the  $W$  boson and represents a background to searches for new physics beyond the standard model. Moreover, it represents a contribution to the  $e^+e^- \rightarrow W^+W^-$  total cross section, used to derive a value for  $M_w$ , the  $W$  boson mass. The issue of gauge invariance in the CC20 family has been solved by the introduction of the Fermion-Loop scheme but several subtleties remain, connected with the region of vanishing scattering angle of the electron and with the limit of massless final state fermions in a fully extrapolated setup. A satisfactory solution for computing the total cross section is given in the context of the equivalent photon or Weizsäcker-Williams approximation which factorizes the flux of quasi-real photons emitted by the electron from the interaction rate between the positron and the photon assumed to be real. The correct kinematics for the inclusion of initial state QED radiation is established. QCD corrections to the process are discussed and numerical results are shown and commented.

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# 1 Introduction

The so-called single  $W$  production process is  $e^+e^- \rightarrow e^-\bar{\nu}_e W^+$  for small scattering angles of the outgoing electron and it has been measured at LEP 2 at centre-of-mass energies  $130 \text{ GeV} \leq \sqrt{s} \leq 183 \text{ GeV}$  using both leptonic and hadronic decays of  $W$  bosons [1]. The signal is, therefore, defined as

$$\begin{aligned} e^+e^- &\rightarrow e^-\bar{\nu}_e l^-\bar{\nu}_l, \\ e^+e^- &\rightarrow e^-\bar{\nu}_e u\bar{d}, \end{aligned} \quad (1)$$

where  $u(d)$  stands for a generic up-(down-)quark. In the terminology of [2] it is a four-fermion process belonging to the CC20-family. The charge conjugate reactions are always understood to be included. The cross section for single  $W$  production is expected to be small at LEP 2 energies, of the order of 0.5 pb. However, this process constitutes a very interesting case both theoretically and experimentally. It is a sensitive probe of anomalous electromagnetic couplings of the  $W$  boson and represents a background to searches for new physics beyond the standard model.

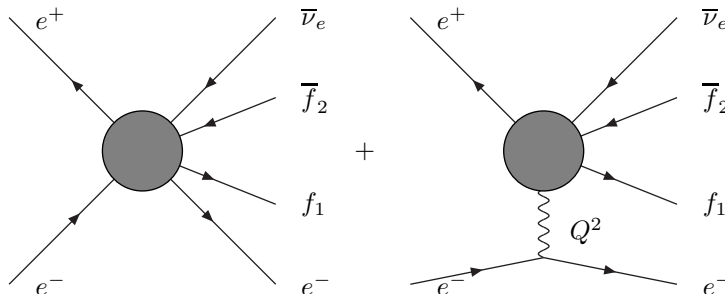


Figure 1: The CC20 family of diagrams with the explicit component containing a  $t$ -channel photon.

The CC20 process of Fig. 1 is sensitive to the breaking of  $U(1)$  gauge invariance in the collinear limit. For  $e^+e^- \rightarrow e^-\bar{\nu}_e f_1\bar{f}_2$ , the  $U(1)$  gauge invariance becomes essential in the region of phase space where the angle between the incoming and outgoing electrons is small, see the work of [3] and also alternative formulations in [4, 5]. In this limit the superficial  $1/Q^4$  divergence of the propagator structure is reduced to  $1/Q^2$  by  $U(1)$  gauge invariance. In the presence of light fermion masses this gives rise to the familiar  $\ln(m_e^2/s)$  large logarithms. The correct way of handling CC20 is represented by the so-called Fermion-Loop (FL)scheme [3], the gauge-invariant treatment of the finite-width effects of  $W$  and  $Z$  bosons in LEP2 processes. Briefly, this scheme consists in including all fermionic one-loop corrections in tree-level amplitudes and re-summing the self-energies. However, for practical applications, it has been shown that the fixed-width scheme is also satisfactory. Here the cross-section is computed using the tree-level amplitude. The massive gauge-boson propagators are given

by  $1/(p^2 + m^2 - i\Gamma m)$ . This gives an unphysical width for  $p^2 > 0$ , but retains U(1) gauge invariance in the CC20 process [6].

The CC20 process is usually considered in two regimes,  $|\cos\theta(e^-)| \geq c$  or LACC20 and  $|\cos\theta(e^-)| \leq c$  or SACC20. Strictly speaking the single  $W$  production is defined by those events that satisfy  $|\cos\theta(e^-)| \geq 0.997$  and, therefore is a SACC20.

The LACC20 cross section has been computed by many authors and references can be found in [2]. It represents a contribution to the  $e^+e^- \rightarrow W^+W^-$  total cross section, in turn used to derive a value for  $M_W$ , the  $W$  boson mass. This point deserves a comment: by  $e^+e^- \rightarrow W^+W^-$  it is meant the ideal cross section obtained with the three double-resonant CC03 diagrams and therefore the background, FULL - CC03, is evaluated with the help of some MonteCarlo, estimating the error on the subtraction by comparing with some other MonteCarlo. Then  $M_W$  is derived from a fit to  $\sigma(\text{CC03})$  with the help of a third calculation. From a theoretical point of view the evaluation of LACC20 is free of ambiguity, even in the approximation of massless fermions, as long as a gauge-preserving scheme is applied and  $\theta(e^-)$  is not too small.

For SACC20 instead, one cannot employ the massless approximation anymore and this fact makes the calculation unaccessible to most of the MonteCarlos used by the experimental collaborations, with the noticeable exception of GRC4F [7]. EXCALIBUR [8] is often used in this context with a version where a fudge is put so that for one-electron final states one can go down to zero scattering angle.<sup>1</sup>

Actually, constructing a CC20 calculation with unconstrained electron scattering angle is not a problem from the point of view of writing the fully massive amplitudes but it is, instead, a question of stability in the numerical integration. Moreover the goal of this paper is to show that several subtleties arise in CC20 for a fully extrapolated setup.

Single  $W$  production and  $M_W$  measurement are, therefore, complementary. Indeed, the phase space requirement  $|\cos\theta(e^-)| > c$  eliminates events predominantly consisting of  $W$  pair production since single  $W$  production peaks strongly at zero scattering angle.

There is another place where the electron angle cannot be constrained. Experimentally events of the type  $e^+e^- \rightarrow u\bar{d}$  plus a neutrino and an electron, possibly in the beam pipe, are not excluded from the hadronic  $Z$  lineshape. Hadronic events are selected based on final state particle multiplicity in the detector, so both genuine high-energy  $q\bar{q}$  events, radiative return events and hadronic four-fermion events are selected for the  $e^+e^- \rightarrow \bar{q}q$  lineshape. This gives the total sample. The background is subtracted on MonteCarlo basis, using  $W$  double-resonant CC03 diagrams, i.e.  $WW \rightarrow \text{all}$ ,  $Z$  double-resonant NC02 diagrams, i.e.  $ZZ \rightarrow \text{all}$  and hadronic two-photon collisions (e.g. using PHOJET [9]).

Therefore double-resonant  $W$ 's, which are dominant, are treated correctly in the experimental procedure and, moreover, single-resonant  $W$ 's only represent

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<sup>1</sup>R. Pittau private communication.

a small contribution. The latter could, however, be treated correctly by using a CC20 MonteCarlo rather than a CC03 one. In this case the electron is again unconstrained and one needs a full angle, massive CC20: the so-called full-CC20, or FCC20, regime.

However, keeping a finite electron mass through the calculation is not enough. One of the main results of this paper is to show that there are subtleties in CC20 also associated with the zero mass limit for the remaining fermions.

The outline of the paper will be as follows. In Sect. 2 we introduce the general problem of defining the total hadronic cross section at LEP2 energies and beyond, and describe how a calculation of FCC20 is to be seen in this context. In Sect. 3 we give a description of FCC20 in the presence of initial state QED radiation and show how to implement the correct kinematics for the process. In Sect. 4 we introduce and discuss the Weizsäcker-Williams approximation for a small scattering angle of the outgoing electron. The sub-process  $e^+\gamma \rightarrow \bar{\nu}_e u \bar{d}$ , arising in the discussion of the WW-approximation is analyzed in Sects. 5-6. The fully extrapolated setup with massless quarks and QCD corrections are presented in Sect. 7. Finally, numerical results and conclusions are shown in Sect. 8.

## 2 The region of vanishing $\theta(e^-)$

There are at least three applications of CC20 which require an analysis at vanishing scattering angle of the outgoing electron,  $\theta(e^-)$ . They are:

1. the true single  $W$  production, i.e. CC20 with  $|\cos\theta(e^-)| \geq c$  where, usually,  $c = 0.997$ ,
2. the evaluation of background for the total  $e^+e^- \rightarrow W^+W^-$  cross section,
3. the evaluation of the inclusive hadronic cross section at LEP2 energies.

Let us consider in more detail the last application. The FCC20 process is not the only background for the total hadronic cross section  $\sigma(\bar{q}qX)$  defined as the cross section for  $\bar{q}q$  plus anything. Here, we would like to illustrate the general problem, to return in the next section to the study of FCC20.

The total hadronic cross section that we have defined is an inclusive measurement of hadron production in  $e^+e^-$ -annihilation in which production thresholds can be seen, e.g.  $W$ -pair production with at least one of the  $W$  bosons decaying hadronically, or  $ZZ$  or other background [10].

Let us repeat what has been done so far in the experimental Collaborations. Hadronic events are selected based on final state particle multiplicity in the detector, so both genuine high energy  $\bar{q}q$  events, radiative returns and four-fermion hadronic events are selected for the hadronic lineshape. This gives the total sample: the background is subtracted on a MonteCarlo, using CC03, NC02 and hadronic two-photon collisions. Clearly the above strategy is good enough for the present precision, but wrong in principle. Let us consider the relation

between (radiatively corrected) two-fermion (2F) and four-fermion (4F) final states in  $e^+e^-$  annihilation at LEP 2. There are several components in the radiative corrections to fermion pair production: among them there is initial-state (or final-state) fermion-pair production. For definiteness consider  $e^+e^- \rightarrow \bar{b}b$  with radiation of an  $e^+e^-$  pair [13]. The background is represented by the full four-fermion process, the so-called NC48 process, which is built out of 48 Feynman diagrams. For studies around the  $Z$  resonance the default [11] was to include pairs from initial state and a cut was selected so that  $M(\bar{b}b) > 0.25$  s. At LEP2 energies or higher one needs a more precise separation between radiative corrections to 2F production and real 4F events [12].

We will denote the evaluation of any one-loop corrected cross section, e.g.  $e^+e^- \rightarrow \bar{b}b$  as a 2F-calculation. By 4F-calculation we mean a tree level evaluation, e.g.  $e^+e^- \rightarrow \bar{b}be^+e^-$ . Note that the soft pairs,  $\gamma^* \rightarrow e^+e^-$  are divergent in the limit of zero  $e^+e^-$  invariant mass and therefore any simulation of very soft pairs with massless 4F-calculations is bound to produce wrong results.

But also a massive 4F-calculation is not enough, because if pairs are soft enough we must include virtual pairs as well, and all  $e^+e^-$  pairs are allowed down to  $M(e^+e^-) = 2m_e$ .

Also soft+virtual initial/final pairs in a 2F-calculation are not enough because no upper cut is imposed on  $M(e^+e^-)$ , so that all pairs compatible with the request  $M(\bar{b}b) > (\text{some thresholds})$  are accepted. Thus there is more than S+V pairs, there are many topologies for hard pairs and some of them require a finite  $m_e$  also for hard pairs. Indeed in NC48 there are multi-peripheral diagrams which diverge for  $m_e \rightarrow 0$ .

The evaluation of  $\sigma(\bar{q}qX)$  requires [13]

- A Include virtual+soft (up to some invariant mass  $\Delta$ ) I/F state pairs with a complete 2F-calculation.
- B The contributions to  $e^+e^- \rightarrow \bar{b}be^+e^-$  not in [A] are then included with a constraint on  $M(\bar{b}b)$ , but with no further restriction on  $M(e^+e^-)$ , with a complete (i.e. fully massive) 4F-calculation.
- C The contributions to  $e^+e^- \rightarrow \bar{b}be^+e^-$  that are already contained in [A], are included with  $M(e^+e^-) > \Delta$ .

Step B requires evaluation of the following 4F-processes: Fully hadronic, with at least one invariant mass passing the cut, semi-leptonic, with  $M(q_j\bar{q}_i)$  passing the cut.

To summarize we may say that around the  $Z$  resonance the rate for real and virtual radiation is known [14] and included in the existing calculations. Both are enhanced by large logarithms but they cancel to a large extent, leading to a small contribution to the inclusive decay rates.

The complete evaluation of  $\sigma(\bar{q}qX)$  would be relatively easy if we could separate sub-classes of diagrams, e.g. primary from secondary production. For that it is necessary that the interference between them be zero, or very small or, at least non-singular. In the limit of massless fermions, singularities will arise from:

1.  $s$ -channel  $\gamma(g)$
2.  $t$ -channel  $\gamma$  with outgoing  $e^\pm$  lost in the beam pipe.

This consideration suggests the appropriate strategy: classes of diagrams showing a mass singularity must be included through some analytical calculation which also accounts for  $\mathcal{O}(\alpha^2)$  virtual radiation, all interferences exhibiting mass singularities belong to this category while the rest, including most of the interferences, are accounted for by some (numerical) massless 4F-calculation.

The goal of this paper is to investigate in more detail the class 2) introduced above, of which CC20 is a prototype.

### 3 Kinematics and structure functions

The inclusion of QED initial state radiation in  $e^+e^-$ -annihilation is based on renormalization group ideas and on factorization of mass singularities. The corresponding cross section may be cast into the following form:

$$\sigma(s) = \int_0^1 dx_1 \int_0^1 dx_2 \Theta_{\text{cut}} \sum_{f=e^+,e^-} D_{e^-}^f(x_1, s) D_{e^+}^{\bar{f}}(x_2, s) \hat{\sigma}_{f\bar{f}}(x_1 x_2 s), \quad (2)$$

where the structure function  $D_e^f(x, s)$  is the probability density to find a *parton*  $f$  with energy fraction  $x$ . The restriction on the region of integration, given by  $\Theta_{\text{cut}}$ , reflects the presence of kinematical cuts.

Let  $p$  be the four-momentum of the incoming electron in the laboratory system,

$$p = \frac{1}{2}\sqrt{s} (0, 0, \beta, 1), \quad \beta^2 = 1 - 4 \frac{m_e^2}{s}. \quad (3)$$

The electron, before interacting, emits soft and collinear photons. Let  $k = k_1 + k_2 + \dots$  be the total four-momentum of the radiated photons. Thus

$$k = \frac{1}{2}\sqrt{s}(1-x) (0, 0, 1, 1), \quad (4)$$

so that  $k^2 = 0$ , as requested by collinear, massless, photons. Usually one can work with the massless approximation for the electron taking part in the hard scattering, thus an on-shell (massless) electron can emit a bunch of massless, collinear, photons and remain on its (massless) mass shell. But the electron mass cannot be neglected in the hard CC20 scattering and, after radiation, the electron finds itself in a virtual state having four-momentum

$$\hat{p} = p - k = \frac{1}{2}\sqrt{s} (0, 0, \beta - 1 + x, x), \quad (5)$$

with  $x$  being the fraction of energy remaining after radiation. As a consequence, the electron is put off its mass shell,

$$\hat{p}^2 = -m_e^2 + \frac{1}{2}(1-\beta)(1-x)s \sim -x m_e^2 \quad \text{for } m_e \rightarrow 0. \quad (6)$$

When considering the whole process we introduce  $p_{\pm}$  for the incoming  $e^{\pm}$  in the laboratory system. Once radiation has been emitted the momenta will be denoted by  $\hat{p}_{\pm}$  with

$$\hat{p}_{\pm} = \frac{1}{2}\sqrt{s} (0, 0, \mp(\beta - 1 + x_{\pm}), x_{\pm}). \quad (7)$$

The total four-momentum becomes

$$\hat{P} = \hat{p}_+ + \hat{p}_- = \frac{1}{2}\sqrt{s} (0, 0, x_- - x_+, x_- + x_+), \quad (8)$$

with a corresponding invariant mass

$$\hat{P}^2 = -x_+x_-s = \hat{s} \quad (9)$$

In the following we will be able to discuss the effects of a correct treatment of QED initial-state radiation (ISR) on the processes under consideration.

## 4 Weizsäcker-Williams approximation for CC20

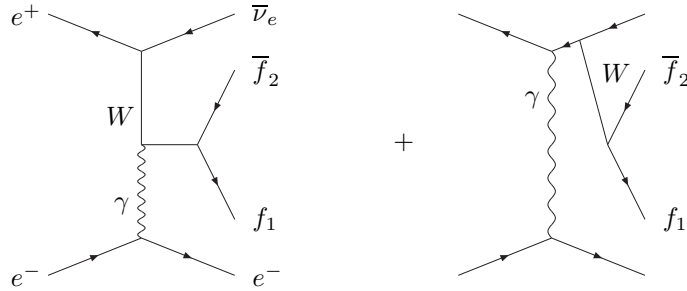
The strategy for the calculation of the CC20 process will be as follows. First, we split the 20 Feynman diagrams of the CC20 family into the four diagrams of Fig. 2, characterized by the presence of a  $t$ -channel photon, and the rest

$$\text{CC20} = \text{CC20}_{\gamma} + \text{CC20}_{\text{R}}. \quad (10)$$

Then we introduce  $\theta_c$ , the angle separating the SACC20 from the LACC20 regions. The total cross section will be computed as

$$|\text{CC20}_{\gamma}^{\leq}(m_e)|^2 + |\text{CC20}^>(0)|^2 + 2 \left[ \text{CC20}_{\gamma}^{\leq}(0) \right]^{\dagger} \text{CC20}_{\text{R}}^{\leq}(0) + |\text{CC20}_{\text{R}}^{\leq}(0)|^2 \quad (11)$$

where  $\text{CC20}^>(\text{CC20}^{\leq})$  implies  $\theta > \theta_c(\theta < \theta_c)$  and the argument  $m_e(0)$  implies a finite(zero) electron mass. For the first term in Eq.(11) we need an analytical calculation which keeps  $m_e \neq 0$  while the remaining terms can be treated numerically with the approximation of  $m_e = 0$ . The square of the four diagrams of Fig. 2 will be computed within the improved Weizsäcker-Williams - approximation (WW), provided that  $\theta_c$  is not too large. This application of the WW-approximation is very similar to the one applied in [15].



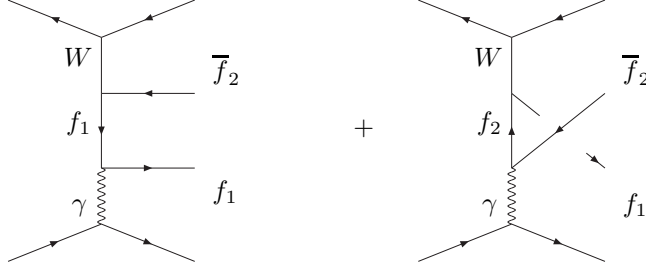


Figure 2: The CC20 $_{\gamma}$  family of diagrams.

The advantage of using the improved WW-approximation is in the possibility of performing an analytical integration over the momentum transferred to the photon, allowing to obtain the exact logarithmic enhancement as well as the first, constant, correction to it.

The kernel cross section for the process CC20 $_{\gamma}(m_e)$ ,

$$e^+(\hat{p}_+)e^-(\hat{p}_-) \rightarrow e^-(q_-)\bar{\nu}_e(q_+)u(k)\bar{d}(\bar{k}), \quad (12)$$

can be written as

$$\hat{\sigma} = \frac{g^2 s_\theta^2}{(2\pi)^8} \frac{N_c}{2\hat{s}} \int d^4 q_- \delta^+(q_-^2 + m_e^2) \int d\Phi_3 \frac{1}{\hat{Q}^4} \hat{L}_{\mu\nu} \hat{W}_{\mu\nu}, \quad (13)$$

where  $\hat{Q} = \hat{p}_- - q_-$  and  $N_c = 1$  for a fully leptonic final state and 3 otherwise. Furthermore  $s_\theta$  is the sine of the weak mixing angle. In this equation,  $d\Phi_3$  is the phase space integral for the  $\bar{\nu}_e u \bar{d}$  system,  $\delta^+(p^2 + m^2) = \theta(E)\delta(p^2 + m^2)$  and  $\hat{L}, \hat{W}$  are the leptonic tensor and the SACC20 tensor. A straightforward calculation gives

$$\hat{L}_{\mu\nu} = \frac{1}{2} \left[ \hat{Q}^2 - (1 - x_-) m_e^2 \right] \delta_{\mu\nu} + \hat{p}_{-\mu} q_{-\nu} + \hat{p}_{-\nu} q_{-\mu}. \quad (14)$$

The four diagrams of Fig. 2 form a  $U(1)$  gauge-invariant set and therefore  $\hat{Q}_\mu \hat{W}_{\mu\nu} = \hat{Q}_\nu \hat{W}_{\mu\nu} = 0$ . The  $\hat{W}$ -tensor admits a decomposition into three form factors

$$\begin{aligned} \int d\Phi_3 \hat{W}_{\mu\nu} &= \hat{W}_1 \left( -\delta_{\mu\nu} + \frac{\hat{Q}_\mu \hat{Q}_\nu}{\hat{Q}^2} \right) - \hat{W}_2 \frac{\hat{Q}^2}{(\hat{p}_+ \cdot \hat{Q})^2} \mathcal{P}_\mu \mathcal{P}_\nu + \hat{W}_3 \varepsilon_{\mu\nu\alpha\beta} \frac{\hat{Q}^\alpha \hat{p}_+^\beta}{\hat{p}_+ \cdot \hat{Q}}, \\ \mathcal{P}_\mu &= \hat{p}_{+\mu} - \frac{\hat{p}_+ \cdot \hat{Q}}{\hat{Q}^2} \hat{Q}_\mu. \end{aligned} \quad (15)$$

Note that the  $\hat{W}_3$  form factor gives zero contribution in this case. Actually the gauge invariance of CC20 $_{\gamma}$  poses a problem with a well-known solution: a



complete treatment would require the application of the Fermion-Loop scheme, but for our purposes it is enough to introduce a fixed width for the  $W$ , both for the  $s$ -channel and the  $t$ -channel, i.e. the fixed-width scheme. For a complete discussion we refer to the work in [3].

In the limit  $\hat{Q}^2 \rightarrow 0$ ,  $\hat{W}^{\mu\nu}$  must be an analytical function of  $\hat{Q}^2$ . By requiring that  $\hat{Q}^2 \int d\Phi_3 \hat{W}^{\mu\nu} = 0$  for  $\hat{Q}^2 = 0$  one obtains

$$\hat{W}_2(\hat{Q}^2, \hat{y}) = \hat{W}_1(0, \hat{y}) + \mathcal{O}(\hat{Q}^2), \quad (16)$$

where we have introduced the variable

$$\hat{y} = \frac{\hat{p}_+ \cdot \hat{Q}}{\hat{p}_+ \cdot \hat{p}_-}. \quad (17)$$

The WW-approximation is defined by the following equation:

$$\begin{aligned} \int d\Phi_3 \hat{L}_{\mu\nu} \hat{W}_{\mu\nu} &\Rightarrow -\frac{1}{4} \hat{W}_1(0, \hat{y}) f_\gamma(\hat{Q}^2, \hat{y}, x_-) \\ \hat{W}_1(0, \hat{y}) &= -\frac{1}{2} \int d\Phi_3 \hat{W}_{\mu\mu} \Big|_{\hat{Q}^2=0}. \end{aligned} \quad (18)$$

$\hat{W}_1$  is therefore proportional to the cross section for  $e^+\gamma \rightarrow \bar{\nu}_e u \bar{d}$  (with real  $\gamma$ ) and  $f_\gamma$  is the photon density, which in the presence of QED initial state radiation reads as follows:

$$f_\gamma = -m_e^2 \left( 1 + x_- + 2 \frac{1 - x_-}{\hat{y}} \right) + \hat{Q}^2 \left( 1 - \frac{2}{\hat{y}} + \frac{2}{\hat{y}^2} \right). \quad (19)$$

Thanks to Eq.(18) the  $\hat{Q}^2$  integration can be performed analytically. Note that the integrand is the sum of two terms, proportional to

$$\frac{1}{\hat{Q}^2}, \quad \frac{m_e^2}{\hat{Q}^4}. \quad (20)$$

To integrate over  $\hat{Q}^2$  we need the kinematics of the process which is specified, in the laboratory system, by

$$p_- = \frac{1}{2} \sqrt{s} (0, 0, \beta, 1), \quad q_- = E_f (\beta_f \sin \theta, 0, \beta_f \cos \theta, 1), \quad (21)$$

with  $\beta_f^2 = 1 - m_e^2/E_f^2$  and  $\beta$  defined in Eq.(3). Let  $\hat{Q}(Q)$  be the momentum transfer with (without) inclusion of ISR, then

$$\begin{aligned} \hat{Q}^2 &= x_- Q^2 - (1 - x_-) m_e^2 + (1 - \beta)(1 - x_-) E_f \beta_f \sqrt{s} \cos \theta \\ &= x_- Q^2 - (1 - x_-) \left( 1 - 2 \frac{E_f}{\sqrt{s}} \cos \theta \right) m_e^2 + \mathcal{O}(m_e^4/s). \end{aligned} \quad (22)$$

If we introduce the variable  $y$ , equivalent to the fraction of the electron energy carried by the photon in absence of ISR and defined by

$$y = \frac{p_+ \cdot Q}{p_+ \cdot p_-}, \quad (23)$$

then the following relations hold for  $m_e = 0$ ,

$$Q^2 = (1 - \cos \theta) E_f \sqrt{s}, \quad y = 1 - (1 + \cos \theta) \frac{E_f}{\sqrt{s}}. \quad (24)$$

Using this result in Eq.(22) one derives

$$\hat{Q}^2 = x_- Q^2 - (1 - x_-) \left( y + \frac{Q^2}{s} \right) m_e^2 + \mathcal{O} \left( \frac{m_e^4}{s} \right). \quad (25)$$

With ISR one uses  $\hat{y}$  defined in Eq.(17) and the relation between  $\hat{y}$  and  $y$  is obtained from

$$\begin{aligned} 2\hat{p}_+ \cdot \hat{Q} &= -\hat{s} - 2x_+ p_+ \cdot q_- + (x_+ + x_-) m_e^2 + (1 - x_+) \left( y - 1 + \frac{Q^2}{s} \right), \\ 2p_+ \cdot q_- &= (1 - y) (2m_e^2 - s). \end{aligned} \quad (26)$$

For finite electron mass the relations linking  $\hat{Q}^2, \hat{y}$  to  $Q^2, y$  read as follows:

$$\hat{Q}^2 = a Q^2 + b m_e^2 y, \quad \hat{y} = c \frac{Q^2}{s} + d y + e, \quad (27)$$

where the  $a, \dots, e$  coefficients are

$$\begin{aligned} a &= x_- - (1 - x_-) \frac{m_e^2}{s}, & b &= -(1 - x_-), \\ c &= \frac{1 - x_+}{D} \frac{m_e^2}{s}, & d &= \frac{-x_+}{D} + \frac{1 + x_+}{D} \frac{m_e^2}{s}, \\ e &= \frac{1 - x_-}{D} \left( x_+ - \frac{m_e^2}{s} \right), & D &= -x_+ x_- + (x_+ + x_-) \frac{m_e^2}{s}. \end{aligned} \quad (28)$$

We have introduced  $Q^2$  and  $y$  because they are natural variables for describing the outgoing electron in absence of ISR. Indeed, one can show that

$$\frac{d^3 q_-}{E_f} = 2\pi \beta_f E_f dE_f d\cos \theta = \pi \frac{1 + \beta^2}{2\beta} dQ^2 dy. \quad (29)$$

The transition to hatted variables, to be used with ISR, is completed by deriving the jacobian of the transformation,

$$d\hat{Q}^2 d\hat{y} = \left[ 1 + \mathcal{O} \left( \frac{m_e^2}{s} \right) \right] dQ^2 dy. \quad (30)$$

Having specified the relevant variables we now proceed to deriving the boundaries of the phase space. First we derive the boundaries for  $Q^2$ . We start from the relations

$$\begin{aligned} Q^2 &= -2m_e^2 + (1 - \beta\beta_f \cos \theta) E_f \sqrt{s}, \\ y &= 1 - 2 \frac{1 + \beta\beta_f \cos \theta}{1 + \beta^2} \frac{E_f}{\sqrt{s}}, \end{aligned} \quad (31)$$

and introduce a new variable  $\chi$  defined by

$$E_f = \frac{\chi^2 + m_e^2}{2\chi}, \quad \beta_f = \frac{\chi^2 - m_e^2}{\chi^2 + m_e^2}. \quad (32)$$

From Eq.(31) a solution for  $\chi$  is

$$\chi = \frac{1}{2} \frac{1 + \beta^2}{1 + \beta c} \left[ 1 - y + \sqrt{(1 - y)^2 - 4 \frac{1 - \beta^2 c^2}{(1 + \beta^2)^2} \frac{m_e^2}{s}} \right] \sqrt{s}, \quad (33)$$

where  $c = \cos \theta$ . At zero scattering angle for the outgoing electron,  $c = 1$ , one obtains

$$\begin{aligned} \chi(c=1) &= \frac{1}{2} \frac{1 + \beta^2}{1 + \beta} \left[ 1 - y + \sqrt{(1 - y)^2 - 4 \frac{1 - \beta^2}{(1 + \beta^2)^2} \frac{m_e^2}{s}} \right] \sqrt{s} \\ &= 1 - y + \mathcal{O}\left(\frac{m_e^2}{s}\right). \end{aligned} \quad (34)$$

Inserting  $E_f$  from Eq.(32) into Eq.(31) one derives

$$Q^2 = -2m_e^2 + \frac{1}{2} \frac{\sqrt{s}}{\chi} \left[ (1 - \beta c) \chi^2 + (1 + \beta c) m_e^2 \right], \quad (35)$$

and, therefore the lower limit for the square of the momentum transfer is set by

$$\begin{aligned} Q^2(c=1) &= -2m_e^2 + \frac{1}{2} \frac{\sqrt{s}}{\chi} \left[ (1 - \beta) \chi^2 + (1 + \beta) m_e^2 \right] \\ &= m_e^2 \frac{y^2}{1 - y} + \mathcal{O}\left(\frac{m_e^4}{s}\right). \end{aligned} \quad (36)$$

If we now require that  $\theta \leq \theta_c$ , with  $\theta_c \ll 1$ , the limits for  $Q^2$  are as follows:

$$\begin{aligned} Q_0^2 &\leq Q^2 \leq Q_c^2, \\ Q_0^2 &= m_e^2 \frac{y^2}{1 - y}, \quad Q_c^2 = Q_0^2 + \frac{1}{4} \frac{\chi_c^2 - m_e^2}{\chi_c} \theta_c^2 \sqrt{s} + \mathcal{O}\left(\frac{m_e^4}{s}, m_e^2 \theta_c^2\right), \end{aligned} \quad (37)$$

where  $\chi_c$  is

$$\frac{\chi_c}{\sqrt{s}} = \left( 1 - \frac{m_e^2}{s} + \frac{\theta_c^2}{4} \right) (1 - y) + \mathcal{O}\left(\frac{m_e^4}{s^2}, \frac{m_e^2}{s} \theta_c^2\right), \quad (38)$$

giving an upper limit of integration for  $Q^2$

$$Q_c^2 = Q_0^2 + \frac{1}{4} (1 - y) \theta_c^2 s + \mathcal{O}\left(\frac{m_e^4}{s}, m_e^2 \theta_c^2\right). \quad (39)$$

The limits for  $Q^2$  can be immediately translated into limits for  $\hat{Q}^2$ , and one finds

$$\begin{aligned}\hat{Q}_0^2 &\leq \hat{Q}^2 \leq \hat{Q}_c^2, \\ \hat{Q}_0^2 &= m_e^2 (x_- \hat{y} + 1 - x_-) \frac{\hat{y}}{1 - \hat{y}} + \mathcal{O}\left(\frac{m_e^4}{s}\right), \\ \hat{Q}_c^2 &= \hat{Q}_0^2 + \frac{1}{4} x_-^2 (1 - \hat{y}) \theta_c^2 s + \mathcal{O}\left(\frac{m_e^4}{s}, m_e^2 \theta_c^2\right).\end{aligned}\quad (40)$$

The photon flux-function, an essential ingredient of the WW-approximation, is now defined by

$$\mathcal{F}_\gamma = \int_{\hat{Q}_0^2}^{\hat{Q}_c^2} d\hat{Q}^2 \frac{f_\gamma}{\hat{Q}^4}. \quad (41)$$

Next we discuss the limits of integration for  $y$  and assume that the fermions in the final state are massless, apart from the electron. As we will see this can be the origin of new mass singularities. From this point of view ISR is inessential. Let us introduce variables  $\rho$  and  $\kappa$  by

$$\rho = \frac{1 + \beta^2}{2} (1 - y), \quad \kappa = \frac{Q^2 + m_e^2}{s}, \quad (42)$$

such that the electron energy and scattering angle become

$$E_f = \frac{1}{2} (\rho + \kappa) \sqrt{s}, \quad \beta \beta_f \cos \theta = \frac{\rho - \kappa}{\rho + \kappa}. \quad (43)$$

For  $\theta = 0$ , after squaring the second relation in Eq.(43) and substituting  $E_f$  from the first one, one obtains

$$\kappa^2 - 2 \frac{1 + \beta^2}{1 - \beta^2} \rho \kappa - \beta^2 + \rho^2 = 0. \quad (44)$$

In this way the allowed region of the phase space for the outgoing electron is completely specified. It is seen that for  $\rho > \beta$  one has  $\kappa < 0$  or  $Q^2 < -m_e^2$ , i.e. the square of the momentum transfer is not positive definite and crosses the zero independently of the finite electron mass. The variable  $\kappa$  becomes negative for  $\rho > \beta$  or

$$y \leq \frac{(1 - \beta)^2}{1 + \beta^2} \sim 2 \frac{m_e^4}{s^2}. \quad (45)$$

This simple fact is better illustrated by considering the process  $e^+ e^- \rightarrow e^- X$  with  $X = \{\bar{\nu}_e u \bar{d}\}$ . Let the cluster  $X$  be characterized by having four-momentum  $q_X$  and mass  $M$ , i.e.  $q_X^2 = -M^2$ . The  $2 \rightarrow 2$  process  $p_+ + p_- \rightarrow q_- + q_X$  is described in terms of Mandelstam invariants

$$s = -(p_+ + p_-)^2, \quad t = -(p_- - q_-)^2, \quad (46)$$

so that  $Q^2 = -t$  and

$$y = \frac{Q^2 - m_e^2 + M^2}{s - 2m_e^2}. \quad (47)$$

The physical portion of the phase space must satisfy the condition  $X \geq 0$  with

$$X = \frac{1}{4} \lambda(s, m_e^2, m_e^2) \lambda(s, m_e^2, M^2) - s^2 \left[ t - m_e^2 - M^2 + \frac{1}{2} (s + M^2 - m_e^2) \right]^2, \quad (48)$$

where  $\lambda$  is the usual Källén-function. When  $M = 0$  the condition  $X \geq 0$  is equivalent to

$$t_- \leq t \leq t_+, \quad (49)$$

where one easily finds that

$$t_- \sim \frac{27}{4} \frac{m_e^6}{s^2}, \quad t_+ \sim -s, \quad \text{for } m_e \rightarrow 0. \quad (50)$$

Therefore, for  $M = 0$ ,  $t$  is not negative definite. The amplitude squared is proportional to  $1/Q^2$  or to  $m_e^2/Q^4$  and massless quarks induce a singularity, even for finite  $m_e$ , if a cut is not imposed on the invariant mass  $M(u\bar{d})^2$ . The singularity is, in any case, avoided by requiring a cut such that

$$y \geq \frac{(1 - \beta)^2}{1 + \beta^2}. \quad (51)$$

An upper limit on  $y$  is derived by considering again

$$p_+ + p_- \rightarrow q_- + q_X, \quad q_X = q_+ + k + \bar{k} = p_+ + Q. \quad (52)$$

Next we introduce the invariant mass of the quark-antiquark system,

$$M^2 = -(p_+ + Q)^2 = m_e^2 - Q^2 + (s - 2m_e^2)y, \quad (53)$$

and require the constraint

$$\sqrt{s} \geq m_e + M, \quad (54)$$

equivalent to

$$(s - 2m_e^2)y \leq s - 2m_e\sqrt{s} + Q^2. \quad (55)$$

This inequality is satisfied for

$$y \leq 1 - \frac{m_e}{\sqrt{s}}, \quad (56)$$

following from the relation giving  $y_{\max}$  in terms of  $Q_{\min}^2$ ,

$$(s - 2m_e^2)y_{\max} = s - 2m_e\sqrt{s} + Q_{\min}^2, \quad Q_{\min}^2 = Q_0^2. \quad (57)$$

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<sup>2</sup>This fact was firstly pointed out in a private communication of A. Ballestrero.

Here  $Q_0^2$  is taken from Eq.(37). The equivalent bound for  $\hat{y}$  follows as

$$\hat{y} \leq 1 - \frac{m_e}{\sqrt{s}}, \quad (58)$$

With a cut on  $M(\bar{d}u)$  the singularity at  $Q^2 = 0$  (or  $\hat{Q}^2 = 0$  with ISR) is avoided but we still have additional singularities. There are two multi-peripheral diagrams contributing to the CC20 process  $e^+e^- \rightarrow e^-\bar{\nu}_e f_1 \bar{f}_2$ , the last two in Fig. 2. When  $Q^2 = 0$ , i.e. the electron is lost in the beam pipe, and the (massless)  $f_1(f_2)$ -fermion is emitted parallel to the (quasi-real) photon then the internal fermion propagator will produce an enhancement in the cross section. Taking into account a  $\ln m_e^2$  from the photon flux-function, 3 options follow:

1. to consider massive fermions, giving a result proportional to  $\ln m_e^2 \ln m_f^2$ ,
2. to use massless fermions, giving instead  $\ln^2 m_e^2$ ,
3. to introduce an angular cut on the outgoing  $f_1$  and  $\bar{f}_2$  fermions with respect to the beam axis,  $\theta(f_1, \bar{f}_2) \geq \theta_{\text{cut}}$ , giving  $\ln m_e^2 \ln \theta_{\text{cut}}$ .

The first option is clean but ambiguous when the final state fermions are light quarks, what to use for  $m_u, m_d$ ? The second one presents no problems for a fully leptonic CC20 final state but completely fails to describe quarks, as we will show in discussing QCD corrections. The last option is also theoretically clean and can be used to give differential distributions for the final state jets. It is, however, disliked by the experimentalists when computing the total sample of events: hadronized jets are seen and not isolated quarks. Even if the quark is parallel to the beam axis the jet could be broad enough and the event selected.

These events are also interesting since they correspond to a situation where the electron and one of the quarks are lost in the beam pipe, while the other quark is recoiling against the neutrino, i.e. one has a totally imbalanced mono-jet structure, background to new particle searches.

The singularity induced by massless quarks in  $e^+e^- \rightarrow e^-\bar{\nu}_e u \bar{d}$  can only be treated within the context of QCD final state corrections and of the photon hadronic structure function (PHSF) scenario. We will come back to the problem later in the paper. In the next section we discuss, instead the cross section for the sub-process  $e^+\gamma \rightarrow \bar{\nu}_e u \bar{d}$  in the two regimes, massless and massive quarks. It will be seen that one can use different parametrizations for the corresponding phase space, depending on the presence of kinematical cuts.

## 5 The sub-process $e^+\gamma \rightarrow \bar{\nu}_e u \bar{d}$ with cuts

As a consequence of Eq.(18), the WW-approximation, we write the result for a CC20 cross section as the convolution of the photon flux-function of Eq.(41) with the cross section for  $e^+\gamma \rightarrow \bar{\nu}_e u \bar{d}$ . The process  $e^+(\hat{p}_+)\gamma(\hat{Q}) \rightarrow \bar{\nu}_e(q_+)u(k)\bar{d}(\bar{k})$

is illustrated in Fig. 3 and is described by the following invariants:

$$\begin{aligned}
\hat{p}_+ \cdot \hat{Q} &= -\frac{1}{2} \hat{y} \hat{s}, & \hat{p}_+ \cdot q_+ &= \frac{1}{2} \kappa_+, & \hat{p}_+ \cdot k &= \frac{1}{2} u', & \hat{p}_+ \cdot \bar{k} &= \frac{1}{2} t', \\
\hat{Q} \cdot q_+ &= \frac{1}{2} \kappa_-, & \hat{Q} \cdot k &= \frac{1}{2} t, & \hat{Q} \cdot \bar{k} &= \frac{1}{2} u, \\
q_+ \cdot k &= \frac{1}{2} \zeta_-, & q_+ \cdot \bar{k} &= \frac{1}{2} \zeta_+, \\
k \cdot \bar{k} &= -\frac{1}{2} s'.
\end{aligned} \tag{59}$$

The process takes place at energy  $\sqrt{\hat{y} \hat{s}}$  and there are only five linear-independent invariants, which we choose to be  $\hat{y} \hat{s}$  and  $\tau, x_1, x_2$  and  $z$  defined by

$$t = \tau \hat{y} \hat{s}, \quad s' - \zeta_- = x_1 \hat{y} \hat{s}, \quad s' = x_2 \hat{y} \hat{s}, \quad \kappa_- = z \hat{y} \hat{s}. \tag{60}$$

The phase space can be computed in terms of the following object:

$$\begin{aligned}
\frac{\partial^2 \Phi_3}{\partial x_1 \partial x_2} &= \hat{y}^2 \hat{s}^2 \int d^4 k d^4 \bar{k} d^4 q_+ \delta^+(k^2) \delta^+(\bar{k}^2) \delta^+(q_+^2) \delta^4(\hat{p}_+ + \hat{Q} - k - \bar{k} - q_+) \\
&\times \delta(2k \cdot \bar{k} + x_2 \hat{y} \hat{s}) \delta(2k \cdot \bar{k} + 2q_+ \cdot k + x_2 \hat{y} \hat{s}).
\end{aligned} \tag{61}$$

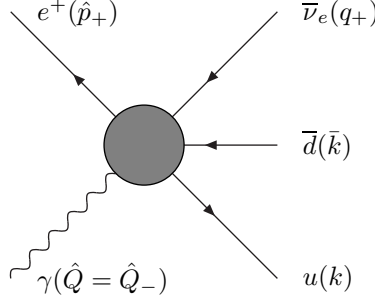


Figure 3: The sub-process  $e^+ \gamma \rightarrow \bar{\nu}_e u \bar{d}$ .

The integration is most conveniently performed in the system where

$$\hat{P} = \hat{p}_+ + \hat{Q} = (0, 0, 0, \sqrt{\hat{y} \hat{s}}). \tag{62}$$

Moreover, let  $\mathbf{k}$  be along the positive  $z$  axis with  $\hat{\mathbf{Q}}$  in the  $x-y$  plane (and polar angle denoted by  $\theta$ ) and let  $\mathbf{q}_+$  be described by angles  $\psi, \phi$ . Then we find,

$$\begin{aligned}
\frac{\partial^2 \Phi_3}{\partial x_1 \partial x_2} &= \hat{y}^2 \hat{s}^2 \theta(x_1) \int d^4 k \delta^+(k^2) \delta \left( 2 E_d \sqrt{\hat{y} \hat{s}} - x_1 \hat{y} \hat{s} \right) J, \\
J &= \theta(1 - x_2) \frac{1 - x_2}{8} \int_{-1}^{+1} d \cos \psi \int_0^{2\pi} d\phi \\
&\times \delta \left( \frac{1}{2} x_1 (1 - x_2) (\cos \psi - 1) \hat{y} \hat{s} + (x_1 - x_2) \hat{y} \hat{s} \right).
\end{aligned} \tag{63}$$

It is more convenient to introduce

$$\begin{aligned}
\frac{\partial^3 \Phi_3}{\partial x_1 \partial x_2 \partial z} &= \hat{y}^3 \hat{s}^3 \theta(x_1) \int d^4 k \delta^+(k^2) \delta \left( 2 E_d \sqrt{\hat{y} \hat{s}} - x_1 \hat{y} \hat{s} \right) \bar{J}, \\
\bar{J} &= \theta(1-x_2) \frac{1-x_2}{8} \int_{-1}^{+1} d \cos \psi \int_0^{2\pi} d\phi \\
&\times \delta \left( ((1-x_2) \sin \theta \sin \psi \cos \phi + (1-x_2)(\cos \theta \cos \psi - 1) - 2z) \hat{y} \hat{s} \right) \\
&\times \delta \left( \frac{1}{2} x_1 (1-x_2) (\cos \psi - 1) \hat{y} \hat{s} + (x_1 - x_2) \hat{y} \hat{s} \right). \tag{64}
\end{aligned}$$

If we now take into account that

$$\cos \theta = 1 + 2 \frac{\tau}{x_1}, \tag{65}$$

the final result follows

$$\begin{aligned}
d\Phi_3 &= \frac{\pi^2 \hat{y} \hat{s}}{4 x_1 R} \Theta dx_1 dx_2 dz d\tau, \\
\Theta &= \left\{ \prod_i \theta(x_i) \theta(1-x_i) \right\} \theta(x_1 - x_2) \theta(z_+ - z) \theta(z - z_-) \theta(-\tau) \theta(\tau + x_1),
\end{aligned} \tag{66}$$

where  $\Theta$  gives the boundaries of the phase space and where we have introduced

$$\begin{aligned}
R^2 &= -4z^2 + 4(1-x_2)(\cos \theta \cos \psi - 1)z - (1-x_2)^2 (\cos \theta - \cos \psi)^2, \\
\cos \psi &= \frac{2x_2 - x_1 - x_1 x_2}{x_1 (1-x_2)}, \tag{67}
\end{aligned}$$

and where  $z_{\pm}$  are the roots of  $R^2 = 0$ , i.e.

$$z_{\pm} = \frac{1-x_2}{2} (\cos \theta \cos \psi \pm |\sin \theta \sin \psi| - 1). \tag{68}$$

The cross section will be computed with a cut on the invariant mass of the  $u\bar{d}$ -pair, i.e.  $M^2(u\bar{d}) \geq s_0$  giving

$$\frac{s_0}{\hat{y} \hat{s}} \leq x_2 \leq 1, \quad \hat{y} \hat{s} \geq s_0. \tag{69}$$

Starting from the four diagrams of Fig. 2 we derive

$$\begin{aligned}
\hat{W} &= -\frac{1}{2} \hat{W}_{\mu\mu} \Big|_{\hat{Q}^2=0} = \frac{g^6 s_\theta^2}{|\Delta_s|^2 |\Delta_t|^2 \hat{s}} \\
&\times \left[ \hat{y}^3 \hat{W}_{11} + |\Delta_t|^2 \hat{W}_{22} + |\Delta_s|^2 \hat{W}_{33} + \hat{y} |\Delta_s|^2 \hat{W}_{44} \right. \\
&+ \hat{y}^2 \left( \hat{W}_{12} + \hat{W}_{13} + \hat{W}_{14} \right) + \hat{y} \left( \hat{W}_{23} + \hat{W}_{24} \right) + \hat{y} |\Delta_s|^2 \hat{W}_{34} \left. \right] \tag{70}
\end{aligned}$$



where the propagators are defined, in the fixed width scheme, by

$$\Delta_s = -x_2 \hat{y} + \mu_w^2 - i \gamma_w \mu_w, \quad \Delta_t = -(1 - x_2 + z) \hat{y} + \mu_w^2 - i \gamma_w \mu_w, \quad (71)$$

with  $\Gamma_w = \gamma_w \sqrt{s}$  and  $\mu_w = M_w \sqrt{s}$ . The  $\hat{W}_{ij}$  represent the  $i - j$  interference of diagrams in Fig. 2 and are given in the following list:

$$\begin{aligned} \hat{W}_{11} &= \tau(1 + zx_1 + 2z - 2x_1x_2 + x_1 + z^2) \\ &+ \tau^2(1 + z - x_2) + zx_1 + zx_1^2 + 2x_1x_2 + x_1z^2 - 2x_2x_1^2 - x_2 + x_1^2, \\ \hat{W}_{22} &= y\tau(1 - x_1), \quad \hat{W}_{33} = \frac{4}{9} \frac{y}{\tau}(1 - x_1), \\ \hat{W}_{44} &= -\frac{1}{9} \frac{\tau}{v}(1 + x_1) + \frac{1}{9} \tau - \frac{1}{9} \frac{\tau^2}{v} - \frac{1}{9} \frac{x_1}{v} + \frac{1}{9} x_1, \\ \hat{W}_{12} &= \text{Re} \Delta_t \left[ \tau(-2 + zx_1 - 2z + 2x_1x_2) \right. \\ &+ \left. \tau^2(-1 + x_2) - zx_1 - 2x_1x_2 + 2x_2x_1^2 + x_2 - x_1^2 \right], \\ \hat{W}_{13} &= \frac{2}{3} \text{Re} \Delta_s \left[ \frac{1}{\tau}(zx_1 - 2zx_1^2 - 2x_1x_2 + x_1 + 2x_2x_1^2 + x_2 - 2x_1^2) \right. \\ &+ \left. \frac{2}{3} \tau(-x_1 + x_2) - \frac{2}{3} - \frac{2}{3} zx_1 + \frac{4}{3} x_1x_2 - \frac{2}{3} x_1^2 \right], \\ \hat{W}_{14} &= \frac{1}{3} \text{Re} \Delta_s \left[ \frac{\tau}{v}(-1 - 2x_1x_2 + 3x_1 - 2x_1^2) + \frac{1}{3} \tau(1 + 2z) \right. \\ &+ \left. \frac{1}{3} \frac{\tau^2}{v}(1 - x_1 - x_2) + \frac{1}{3} \frac{1}{v}(2x_1x_2 - 2x_2x_1^2 - x_2) + \frac{2}{3} zx_1 + \frac{1}{3} x_1^2 \right], \\ \hat{W}_{23} &= -\frac{4}{3} \text{Re} \Delta_s \text{Re} \Delta_t \left[ \frac{x_1}{\tau}(1 - x_1)^2 + \frac{4}{3} x_1 - \frac{4}{3} x_1^2 \right] - \frac{4}{3} \frac{\gamma_w^2 \mu_w^2}{\tau} x_1(1 - x_1)^2 \\ &+ \frac{4}{3} \gamma_w^2 \mu_w^2 x_1(1 - x_1) \\ \hat{W}_{24} &= \frac{1}{3} \text{Re} \Delta_s \text{Re} \Delta_t \left[ \frac{\tau}{v}(1 + 2x_1x_2 + x_1 - 2x_1^2) + \frac{1}{3} \tau(-2 + x_1) + \frac{1}{3} \frac{\tau^2}{v} \right. \\ &\times (1 - x_1 + x_2) + \frac{1}{3} \frac{1}{v}(-2x_1x_2 + 2x_2x_1^2 + x_2 + 2x_1^2 - 2x_1^3) - \frac{1}{3} x_1 \left. \right] \\ &+ \frac{1}{3} \gamma_w^2 \mu_w^2 \frac{\tau}{v}(1 + 2x_1x_2 + x_1 - 2x_1^2) + \frac{1}{3} \gamma_w^2 \mu_w^2 \tau(-2 + x_1) + \frac{1}{3} \gamma_w^2 \mu_w^2 \frac{\tau^2}{v} \\ &\times (1 - x_1 + x_2) + \frac{1}{3} \frac{\gamma_w^2 \mu_w^2}{v}(-2x_1x_2 + 2x_2x_1^2 + x_2 + 2x_1^2 - 2x_1^3) - \frac{1}{3} \gamma_w^2 \mu_w^2 x_1, \\ \hat{W}_{34} &= \frac{2}{9} \frac{1}{\tau v}(-2x_1x_2 + 2x_2x_1^2 + x_2) + \frac{2}{9} \frac{1}{\tau} x_1(1 - 2x_1) \\ &+ \frac{2}{9} \frac{\tau}{v}(-1 + x_1 + x_2) + \frac{2}{9} \frac{1}{v}(1 + 2x_1x_2 - 3x_1 + 2x_1^2) - \frac{2}{9} x_1 \end{aligned} \quad (72)$$

where we have introduced a new variable  $v$ ,

$$v = 1 + \tau + z. \quad (73)$$

There remains the problem of angular cuts for the outgoing quark(antiquark). These cuts should be applied with respect to the beam direction and should be expressed in terms of the variables describing the sub-process. For CC20 $_{\gamma}$  and for  $\theta_c \ll 1$  the  $e^-$  is lost in a narrow cone around the beam, so that we can use the approximation  $\hat{Q} = \hat{y}\hat{p}_-$ . In the laboratory system we have

$$p_{\pm} = \frac{1}{2} \sqrt{s} (0, 0, \mp 1, 1). \quad (74)$$

Let  $E_u$  be the energy of the outgoing  $u$ -quark, so that

$$k = E_u (\sin \theta_u, 0, \cos \theta_u, 1), \quad (75)$$

where  $\theta_u$  is the scattering angle of the  $u$ -quark with respect to the incoming electron. One finds

$$\begin{aligned} \cos \theta_u &= \frac{p_+ \cdot k - p_- \cdot k}{p_+ \cdot k + p_- \cdot k}, \\ p_+ \cdot k &= \frac{\hat{p}_+ \cdot k}{x_+} = \frac{u'}{2x_+}, \quad p_- \cdot k = \frac{\hat{Q} \cdot k}{x_- \hat{y}} = \frac{t}{2x_- \hat{y}}. \end{aligned} \quad (76)$$

The condition  $|\cos \theta_u| \leq C$  becomes, in terms of invariants,

$$\begin{aligned} (1 + C) x_- \hat{y} x_1 + \left[ (1 + C) x_m \hat{y} + (1 - C) x_+ \right] \tau &\leq 0, \\ (1 - C) x_- \hat{y} x_1 + \left[ (1 - C) x_m \hat{y} + (1 + C) x_+ \right] \tau &\geq 0. \end{aligned} \quad (77)$$

With  $\bar{k} = E_d (\sin \theta_d, 0, \cos \theta_d, 1)$  we derive two additional conditions similar to those of Eq.(77) but with  $u' \rightarrow t', t \rightarrow u$  and reflecting the cut  $|\cos \theta_d| \leq C$ .

Eq.(70), in conjunction with Eqs.(13, 18) and Eq.(67), allows us to compute the cross section within the WW-approximation.

## 6 The fully extrapolated sub-process $e^+ \gamma \rightarrow \bar{\nu}_e u \bar{d}$

Our goal is to compute the CC20 cross section without any kinematical cut, apart from imposing that  $M(u\bar{d}) \geq \sqrt{s_0}$ . We have seen in sect. 5 that there is a mass singularity in the total cross section when the quarks are assumed to be massless. Here we compute again the cross section with finite quark masses. Let us consider again the process  $e^+(\hat{p}_+) \gamma(\hat{Q}_-) \rightarrow \bar{\nu}_e(q_+) u(k) \bar{d}(\bar{k})$ . Moreover let  $\hat{Q}_+$  be defined as  $\hat{Q}_+ = \hat{p}_+ - q_+$  and let  $\hat{Q}$  be  $\hat{Q}_+ + \hat{Q}_-$ . In order to compute  $\hat{W}_{\mu\mu}$ , as required by Eq.(18), we neglect for the moment ISR and introduce three master scalar-integrals:

$$I_n = \int d^4 k d^4 \bar{k} \delta^+(k^2 + m_u^2) \delta^+(\bar{k}^2 + m_d^2) \delta^4(Q - k - \bar{k}) \frac{1}{(Q_- \cdot k)^n}, \quad (78)$$

with  $n = 0, 2$ . They are easily evaluated in the system where  $Q = (0, 0, 0, \mu)$  and where the photon four-momentum is  $Q_- = X(0, 0, 1, 1)$ . One immediately finds

$$X = \frac{Q_+ \cdot Q_-}{Q^2}. \quad (79)$$

If we start with  $I_2$  then we obtain

$$\begin{aligned} I_2 &= \int d^4k \delta^+(k^2 + m_u^2) \delta^+(2\mu E_u - \mu^2 - m_u^2 + m_d^2) \frac{1}{(Q_- \cdot k)^2} \\ &= \frac{\pi}{4\mu X^2} \int_0^\infty k \delta(k^2 - E_u^2 + m_u^2) \frac{2k}{m_u^2} \\ &= \frac{\pi}{4} \frac{E}{\mu X^2 m_u^2} \sim -\frac{\pi}{8} \frac{Q^2}{(Q_+ \cdot Q_-)^2} \frac{1}{m_u^2} \quad \text{for } m_u \rightarrow 0. \end{aligned} \quad (80)$$

Similarly we evaluate  $I_1$  as follows:

$$\begin{aligned} I_1 &= \int d^4k \delta^+(k^2 + m_u^2) \delta^+(2\mu E_u - \mu^2 - m_u^2 m_d^2) \frac{1}{X(k \cos \theta - E_u)} \\ &= -\frac{\pi}{2\mu X} \int_0^\infty dk k \delta(k^2 + m_u^2) \ln \frac{E_u - k}{E_u + k} \\ &= -\frac{\pi}{2\mu X} \ln \frac{m_u}{E_u + \sqrt{E_u^2 - m_u^2}} \sim \frac{\pi}{4Q_+ \cdot Q_-} \ln \frac{m_u^2}{-Q^2} \quad \text{for } m_u \rightarrow 0 \end{aligned} \quad (81)$$

Finally, for  $I_0$  one gets

$$I_0 = \frac{\pi}{2}. \quad (82)$$

The complete result for the cross section follows from squaring the matrix element,

$$W_{\mu\nu} = \frac{g^6 s_\theta^2}{64} \mathcal{R}_\nu^\dagger (-i \not{k} + m_u) \mathcal{R}_\mu (i \not{\bar{k}} + m_d) \quad (83)$$

The function  $\mathcal{R}$  is

$$\begin{aligned} \mathcal{R}_\mu &= \gamma^\alpha \gamma_+ \mathcal{R}_{\mu\alpha}^1 P(s)P(t) + \gamma^\alpha \gamma_+ \mathcal{R}_{2\mu\alpha} P(s)P(e) \\ &+ \gamma_\mu (\hat{Q} - \not{k} - i m_u) \gamma^\alpha \gamma_+ \mathcal{R}_\alpha^3 P(t)P(f) \\ &+ \gamma^\alpha \gamma_+ (\not{Q}_- - \not{\bar{k}} + i m_d) \gamma_\mu \mathcal{R}_\alpha^4 P(t)P(\bar{f}). \end{aligned} \quad (84)$$

Furthermore we have  $\gamma_+ = 1 + \gamma_5$  and

$$\begin{aligned} \mathcal{R}_{\mu\alpha}^1 &= V_{\mu\beta\alpha} \bar{v}(p_+) \gamma^\beta \gamma_+ v(q_+), \\ \mathcal{R}_{\mu\alpha}^2 &= -\bar{v}(p_+) \gamma_\mu (\not{p}_+ + \not{Q}_-) \gamma_\alpha \gamma_+ v(q_+), \\ \mathcal{R}_\mu^3 &= -Q_u \bar{v}(p_+) \gamma_\mu \gamma_+ v(q_+), \\ \mathcal{R}_\mu^4 &= Q_d \bar{v}(p_+) \gamma_\mu \gamma_+ v(q_+), \end{aligned} \quad (85)$$

where  $V_{\mu\alpha\beta}$  is the corresponding triple gauge-boson vertex,  $Q_u(Q_d)$  is the up-(down-)fermion charge and the propagators appearing in Eq.(84) are

$$\begin{aligned} P(s) &= \frac{1}{(k + \bar{k})^2 + M_W^2 - i\Gamma_W M_W}, & P(t) &= \frac{1}{(p_+ - q_+)^2 + M_W^2 - i\Gamma_W M_W}, \\ P(e) &= \frac{1}{(p_+ - Q_-)^2 + m_e^2}, \\ P(f) &= \frac{1}{(Q_- - k)^2 + m_u^2}, & P(\bar{f}) &= \frac{1}{(Q_- - \bar{k})^2 + m_d^2}. \end{aligned} \quad (86)$$

Next, we have to integrate over the phase space. The integration over  $k, \bar{k}$  can be performed by introducing additional integrals:

$$\begin{aligned} I_{n,l}^{\mu_1 \dots \mu_n}(u) &= \int d^4k d^4\bar{k} \delta^+(k^2 + m_u^2) \delta^+(\bar{k}^2 + m_d^2) \delta^4(Q - k - \bar{k}) \\ &\quad \times \frac{k^{\mu_1} \dots k^{\mu_n}}{(Q_- \cdot k)^l}, \\ I_{n,l}^{\mu_1 \dots \mu_n}(d) &= \int d^4k d^4\bar{k} \delta^+(k^2 + m_u^2) \delta^+(\bar{k}^2 + m_d^2) \delta^4(Q - k - \bar{k}) \\ &\quad \times \frac{\bar{k}^{\mu_1} \dots \bar{k}^{\mu_n}}{(Q_- \cdot \bar{k})^l}. \end{aligned} \quad (87)$$

All these integrals can be reduced to the scalar form factors. Quarks masses are kept only in front of  $I_2$  and the reduction gives

$$\begin{aligned} I_{3n}^{\mu\nu\alpha}(q) &= Q^\mu Q^\nu Q^\alpha I_{3n,31}(q) + Q_-^\mu Q_-^\nu Q_-^\alpha I_{3n,32}(q) + (Q_-^\mu Q^\nu Q^\alpha + Q^\mu Q_-^\nu Q^\alpha \\ &\quad + Q^\mu Q^\nu Q_-^\alpha) I_{3n,33}(q) + (Q^\mu Q_-^\nu Q_-^\alpha + Q_-^\mu Q^\nu Q^\alpha + Q_-^\mu Q_-^\nu Q^\alpha) I_{3n,34}(q) \\ &\quad + (Q^\mu \delta^{\nu\alpha} + Q^\nu \delta^{\mu\alpha} + Q^\alpha \delta^{\mu\nu}) I_{3n,35}(q) \\ &\quad + (Q_-^\mu \delta^{\nu\alpha} + Q_-^\nu \delta^{\mu\alpha} + Q_-^\alpha \delta^{\mu\nu}) I_{3n,36}(q) \\ I_{30}^{\mu\nu\alpha} &= \frac{I_0}{4} \left[ Q^\mu Q^\nu Q^\alpha - (Q^\mu \delta^{\nu\alpha} + Q^\nu \delta^{\mu\alpha} + Q^\alpha \delta^{\mu\nu}) \frac{Q^2}{6} \right] \\ I_{2n}^{\mu\nu}(q) &= Q^\mu Q^\nu I_{2n,21}(q) + Q_-^\mu Q_-^\nu I_{2n,22}(q) + (Q^\mu Q_-^\nu + Q_-^\mu Q^\nu) I_{2n,23}(q) \\ &\quad + \delta^{\mu\nu} I_{2n,24}(q) \\ I_{20}^{\mu\nu} &= \frac{I_0}{3} \left( Q^\mu Q^\nu - \delta^{\mu\nu} \frac{Q^2}{4} \right), \quad I_{10}^\mu = Q^\mu \frac{I_0}{2} \\ I_{1n}^\mu(q) &= Q^\mu I_{1n,11}(q) + Q_-^\mu I_{1n,12}(q), \end{aligned} \quad (88)$$

where  $q = u, d$  and where all the form factors can be reduced to linear combinations of the master scalar integrals of Eq.(78). After a straightforward algebra

one obtains

$$\begin{aligned}
I_{12,11}(q) &= 0, \\
I_{12,12}(q) &= I_2(q) \left[ 1 + \frac{1}{2} \frac{Q_+^2}{Q_+ \cdot Q_-} \right], \\
I_{11,11}(q) &= \frac{I_0}{Q_+ \cdot Q_-}, \\
I_{11,12}(q) &= \left[ I_1(q) - 2 \frac{I_0}{Q_+ \cdot Q_-} \right] \left[ 1 + \frac{1}{2} \frac{Q_+^2}{Q_+ \cdot Q_-} \right], \\
I_{22,24}(q) &= 0, \quad I_{22,21}(q) = 0, \quad I_{22,23}(q) = 0, \\
I_{22,22}(q) &= I_2(q) \left[ 1 + \frac{1}{2} \frac{Q_+^2}{Q_+ \cdot Q_-} \right]^2, \\
I_{21,24}(q) &= -\frac{1}{2} I_0 \left[ 1 + \frac{1}{2} \frac{Q_+^2}{Q_+ \cdot Q_-} \right], \\
I_{21,21}(q) &= \frac{1}{2} \frac{I_0}{Q_+ \cdot Q_-}, \\
I_{21,23}(q) &= \frac{1}{2} \frac{I_0}{Q_+ \cdot Q_-} \left[ 1 + \frac{1}{2} \frac{Q_+^2}{Q_+ \cdot Q_-} \right], \\
I_{21,22}(q) &= \left[ I_1(q) - 3 \frac{I_0}{Q_+ \cdot Q_-} \right] \left[ 1 + \frac{1}{2} \frac{Q_+^2}{Q_+ \cdot Q_-} \right]^2, \\
I_{32,36}(q) &= 0, \quad I_{32,35}(q) = 0, \quad I_{32,31}(q) = 0, \quad I_{32,33}(q) = 0, \quad I_{32,34}(q) = 0, \\
I_{32,32}(q) &= I_2(q) \left[ 1 + \frac{1}{8} \frac{Q_+^6}{(Q_+ \cdot Q_-)^3} + \frac{3}{4} \frac{Q_+^4}{(Q_+ \cdot Q_-)^2} + \frac{3}{2} \frac{Q_+^2}{Q_+ \cdot Q_-} \right], \\
I_{31,36}(q) &= -\frac{1}{6} I_0 \left[ 1 + \frac{1}{2} \frac{Q_+^2}{Q_+ \cdot Q_-} \right]^2, \\
I_{31,35}(q) &= -\frac{1}{6} I_0 \left[ 1 + \frac{1}{2} \frac{Q_+^2}{Q_+ \cdot Q_-} \right], \\
I_{31,31}(q) &= \frac{1}{3} \frac{I_0}{Q_+ \cdot Q_-}, \\
I_{31,33}(q) &= \frac{1}{6} \frac{I_0}{Q_+ \cdot Q_-} \left[ 1 + \frac{1}{2} \frac{Q_+^2}{Q_+ \cdot Q_-} \right], \\
I_{31,34}(q) &= \frac{1}{3} \frac{I_0}{Q_+ \cdot Q_-} \left[ 1 + \frac{1}{2} \frac{Q_+^2}{Q_+ \cdot Q_-} \right]^2, \\
I_{31,32}(q) &= I_1(q) \left[ 1 + \frac{1}{8} \frac{Q_+^6}{(Q_+ \cdot Q_-)^3} + \frac{3}{4} \frac{Q_+^4}{(Q_+ \cdot Q_-)^2} + \frac{3}{2} \frac{Q_+^2}{Q_+ \cdot Q_-} \right] \\
&\quad - 11 I_0 \left[ \frac{1}{24} \frac{Q_+^6}{(Q_+ \cdot Q_-)^4} + \frac{1}{4} \frac{Q_+^4}{(Q_+ \cdot Q_-)^3} + \frac{1}{2} \frac{Q_+^2}{(Q_+ \cdot Q_-)^2} \right. \\
&\quad \left. + \frac{1}{3} \frac{1}{Q_+ \cdot Q_-} \right]. \tag{89}
\end{aligned}$$

ISR is restored by changing in the previous equations  $Q, Q_{\pm}$  into  $\hat{Q}, \hat{Q}_{\pm}$ . The kernel cross section for the process, to be convoluted with the  $e^{\pm}$  structure functions, is therefore written as

$$\begin{aligned}\hat{\sigma} &= \frac{g^8 s_{\theta}^4}{(2\pi)^8} \frac{\pi}{64 \hat{s}} \int d\hat{Q}_{-}^2 d\hat{y}_{-} f_{\gamma} \int d\Phi_3 \hat{W}_{\mu\mu} \Big|_{\hat{Q}^2=0}, \\ \int d\Phi_3 \hat{W}_{\mu\mu} \Big|_{\hat{Q}^2=0} &= \frac{\pi^2}{4} N_c \int d\hat{Q}_{+}^2 d\hat{y}_{+} |A|^2,\end{aligned}\quad (90)$$

where  $A$  is the amplitude for the process, function of  $\hat{Q}_{+}^2, \hat{y}_{\pm}$ , with

$$\hat{Q}_{+}^2 = \hat{p}_{+} - q_{+}, \quad \hat{y}_{+} = -2 \frac{\hat{Q}_{+} \cdot \hat{Q}_{-}}{\hat{y}_{-} \hat{s}}, \quad \hat{y}_{-} = \frac{\hat{p}_{+} \cdot \hat{Q}_{-}}{\hat{p}_{+} \cdot \hat{p}_{-}}. \quad (91)$$

Starting from the original  $d\Phi_3$  we have been able to perform the  $k, \bar{k}$  integrations, with the help of Eqs.(87–89), arriving at a twofold,  $d\hat{Q}_{+}^2 d\hat{y}_{+}$ , integral. For the purpose of integration it is more useful to change variable from  $\hat{Q}_{+}^2$  to  $x$ , defined by

$$\hat{Q}_{+}^2 = (\hat{y}_{+} - x) \hat{y}_{-} \hat{s}. \quad (92)$$

The limits of integration and the jacobian of the transformation are:

$$\frac{s_0}{\hat{y}_{-} \hat{s}} \leq x \leq \hat{y}_{+}, \quad \frac{s_0}{\hat{y}_{-} \hat{s}} \leq \hat{y}_{+} \leq 1, \quad d\hat{Q}_{+}^2 d\hat{y}_{+} = \hat{y}_{-} \hat{s} d\hat{y}_{+} dx. \quad (93)$$

Before giving the complete expression for  $|A|^2$  and computing the cross sections we have to answer the question of what to do with the light quark masses. The following section is devoted to a clarification of the origin of these additional mass singularities.

## 7 QCD corrections

We have already indicated that, for massless quarks, the cross section for  $e^{+}e^{-} \rightarrow e^{-}\bar{\nu}_e u \bar{d}$  is dominated by two large logarithms. One originates in the limit of small scattering angle of the outgoing electron. The other comes from the propagator of the internal light quark in the multi-peripheral diagrams. Another way of looking at it is to reconsider the integral  $I_1$  of Eq.(81) and to evaluate it for  $m_u = m_d = 0$  and arbitrary  $Q_{\pm}^2$ . One obtains

$$\begin{aligned}I_1^{(0)} &= \int d^4k d^4\bar{k} \delta^{+}(k^2) \delta^{+}(\bar{k}^2) \delta^4(Q - k - \bar{k}) \frac{1}{(Q_{-} - k)^2} \\ &= \frac{\pi}{4\sqrt{\Delta}} \ln \frac{Q_{+} \cdot Q_{-} - \sqrt{\Delta}}{Q_{+} \cdot Q_{-} + \sqrt{\Delta}},\end{aligned}\quad (94)$$

with  $\Delta$  being a Gram's determinant,

$$\Delta = (Q_{+} \cdot Q_{-})^2 - Q_{+}^2 Q_{-}^2. \quad (95)$$

Therefore, for small scattering angles of the outgoing electron, the integral behaves like

$$I_1^{(0)} \sim \frac{\pi}{4 Q_+ \cdot Q_-} \ln \frac{Q_+^2 Q_-^2}{4 (Q_+ \cdot Q_-)^2}, \quad \text{for } Q_-^2 \rightarrow 0. \quad (96)$$

Note that  $(Q_- - k)^2$  appears in the internal quark propagator of the multiperipheral diagrams. For values of  $Q_-^2$  small enough, the lower limit for  $(Q_- - k)^2$  becomes much smaller than  $\Lambda_{QCD}$ , well beyond the limit of applicability of perturbative QCD. This fact has many similarities to the inelastic  $ep$  scattering, see the work in [16].

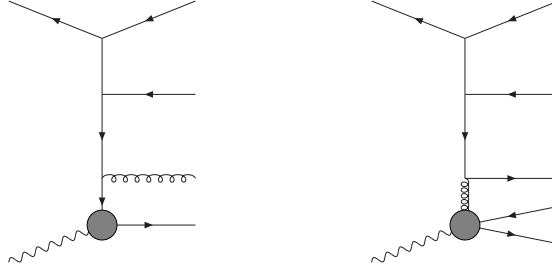


Figure 4: Example of resolved type processes in CC20 $_{\gamma}$ .

So far, QCD corrections to the CC20 process have been applied within the context of naive QCD [2] or with a complete  $\mathcal{O}(\alpha_s)$  calculation [17] which assumes a point-like coupling of the photon to quarks. However, the large logarithm of Eq.(96) receives contributions from any order in  $\alpha_s$  from multiple gluon radiation. The latter creates a series of extra quark propagators, each yielding an extra power of the logarithm compensating the additional power of  $\alpha_s$ . This fact is discussed in [16] and in [18]. Therefore, logarithmically enhanced terms, of order  $\alpha_s^n \ln^n(m_q^2/Q^2)$  appear at every order in the perturbative expansion and, since the logarithm is large, the perturbative series does not converge quickly. Fortunately, this difficulty can be obviated, at least in principle. A formalism exists to sum these logarithms to all orders in perturbation theory, see [19].

The point is that a photon has a point-like coupling to the quark-antiquark pair only for sufficiently high virtuality. On the contrary, for small electron scattering angle, the photon is quasi-real in our CC20 process and behaves like a meson. This fact and its consequences are well-known in other processes, like the total  $\gamma p$  cross section. At low photon virtualities one also expects contributions from the partonic constituents of the photon. The two contributions form the so-called resolved photon component, see Fig. 4, which is usually added to the direct one, computed to fixed order in perturbation theory, where the photon is treated as an elementary particle. The resolved photon component is given

in terms of the photon, hadronic, structure function. There is a well-known subtlety in adding direct and resolved components. The direct component, evaluated at some fixed order in  $\alpha_s$  from all corresponding Feynman diagrams, contains singular terms that are already re-summed in the resolved component. The correct result [16] is schematically represented in the following equation:

$$\hat{\sigma} = \int d\hat{y}_- \mathcal{F}_\gamma \left[ \hat{\sigma}_{\gamma e}^{\text{res}} + \hat{\sigma}_{\gamma e}^{\text{dir,sub}} \right], \quad (97)$$

where the superscript *sub* indicates that one must subtract the terms responsible for the large logarithms in the direct photon component and where  $\hat{\sigma}_{\gamma e}$  is the direct or the resolved cross section for  $\gamma e^+ \rightarrow \bar{\nu}_e u \bar{d}$  at fixed  $\hat{y}_-$ . In the resolved cross section the photon interacts via the quark or the gluon component in its structure function. Therefore one has [18]

$$\hat{\sigma}_{\gamma e}^{\text{res}} = \sum_{i=q,g} \int d\eta F_{i\gamma}(\eta, M) \hat{\sigma}_{ie \rightarrow \text{jet}}. \quad (98)$$

here  $F_{i\gamma}$  is the PHSF and  $M$  is the factorization scale. As shown in Eq.(97) the cross section for the CC20 process,  $e^+e^- \rightarrow e^- \bar{\nu}_e \text{jet}$  follows from the cross section for  $e^+\gamma \rightarrow \bar{\nu}_e \text{jet}$  by applying the equivalent photon or Weizsäcker-Williams approximation which factorizes the flux of quasi-real photons emitted by the  $e^-$  from the interaction rate between the positron and the photon assumed to be real.

The introduction of a resolved component for the photon is a familiar topic in  $\gamma p$  or  $\gamma\gamma$  scattering. Here the situation is slightly different. The small virtuality of the photon is only needed when CC20 is a background to the high-energy hadronic lineshape or to Higgs boson searches or for single  $W$  production. The cross section for CC20 $_\gamma$  is obtained starting from four Feynman diagrams, two being single  $W$  resonant and two being multi-peripheral. In single  $W$  production, where one applies a cut  $|\cos\theta(e^-)| \geq 0.997$ , the QCD corrections are important and it appears difficult to obtain a precise prediction for the total cross section without summing the large logarithms into the  $F_{i\gamma}$  distribution function.

For the hadronic lineshape, on the other end, hadronic events are selected based on final state particle multiplicity in the detector. This gives the total sample but more interesting is the high-energy  $M^2(u\bar{d}) \geq s_0$  sample. The relative dominance of the multi-peripheral diagrams in CC20 $_\gamma$  is larger in the total sample but not necessarily in the high-energy one. Therefore the uncertainty associated with the use of the PHSF calculated at the zeroth order in  $\alpha_s$ , i.e. in the Born approximation, is less relevant if we apply a strong invariant mass cut.

Our strategy, for the moment, will be to use the parton model result, i.e. zeroth order in  $\alpha_s$ , and to cure the ill-defined massless limit by replacing the quark masses with a factorization scale  $M$ , in our case  $m_u = m_d \rightarrow M$ . The total cross section for the CC20 process will, therefore, depend on the scale  $M$ .



The amplitude squared, to be inserted in Eq.(90), becomes

$$\begin{aligned}
|A|^2 = & \frac{4}{3} \frac{\hat{y}_-}{|\Delta_s|^2} v^2 \gamma_w^2 \mu_w^2 \frac{x}{Y} \frac{1-\hat{y}_+}{\hat{y}_+^4} \left[ \hat{y}_+^2 + x(-3\hat{y}_+ + 2x) \right] \\
& + \frac{1}{3} \frac{\hat{y}_-}{|\Delta_s|^2} v^2 \frac{x}{\hat{y}_+^2} \left[ 2 - 4\hat{y}_+ - \hat{y}_+^3 + \hat{y}_+^2 + 2x(1-\hat{y}_+)(-\hat{y}_+ + x) \right] \\
& + \frac{2}{3} \frac{\hat{y}_-^2}{|\Delta_s|^2} v^3 \mu_w^2 \frac{x}{Y} \frac{1-\hat{y}_+}{\hat{y}_+^3} \left[ \hat{y}_+^2 + x(-3\hat{y}_+ + 2x) \right] \\
& + \frac{2}{3} \frac{\hat{y}_-^3}{|\Delta_s|^2} v^4 \frac{x^2}{Y} \frac{1-\hat{y}_+}{\hat{y}_+^3} \left[ -\hat{y}_+^2 + x(3\hat{y}_+ - 2x) \right] \\
& + \frac{1}{9} \frac{\hat{y}_-}{|\Delta_t|^2} L \frac{v^2}{\hat{y}_+^4} \left[ -5\hat{y}_+^3 - \hat{y}_+^5 + 2\hat{y}_+^4 + x(15\hat{y}_+^2 - 6\hat{y}_+^3 + 3 - 20x\hat{y}_+ + 8x\hat{y}_+^2 \right. \\
& \left. - 4x\hat{y}_+^3 + 10x^2 - 4x^2\hat{y}_+ + 2x^2\hat{y}_+^2) \right] \\
& + \frac{4}{3} \frac{\hat{y}_-}{|\Delta_t|^2} v^2 \gamma_w^2 \mu_w^2 \frac{x}{Y} \frac{1-\hat{y}_+}{\hat{y}_+^4} \left[ \hat{y}_+^2 + x(-3\hat{y}_+ + 2x) \right] \\
& + \frac{1}{9} \frac{\hat{y}_-}{|\Delta_t|^2} \frac{v^2}{\hat{y}_+^4} \left[ -11\hat{y}_+^3 - \frac{5}{2}\hat{y}_+^5 + 5\hat{y}_+^4 + x(31\hat{y}_+^2 - 19\hat{y}_+^3 + \frac{11}{2}\hat{y}_+^4 - 50x\hat{y}_+ \right. \\
& \left. + 32x\hat{y}_+^2 - 8x\hat{y}_+^3 + 30x^2 - 18x^2\hat{y}_+ + 5x^2\hat{y}_+^2) \right] \\
& + \frac{2}{3} \frac{\hat{y}_-^2}{|\Delta_t|^2} v^3 \mu_w^2 \frac{x}{Y} \frac{1-\hat{y}_+}{\hat{y}_+^3} \left[ -\hat{y}_+^2 + x(3\hat{y}_+ - 2x) \right] \\
& + \frac{2}{3} \frac{\hat{y}_-^3}{|\Delta_t|^2} v^4 \frac{x}{Y} \frac{1-\hat{y}_+}{\hat{y}_+^3} \left[ -\hat{y}_+^3 + x(4\hat{y}_+^2 - 5x\hat{y}_+ + 2x^2) \right]. \tag{99}
\end{aligned}$$

The propagators, in the fixed width scheme, become

$$\Delta_s = \frac{1}{-x\hat{y}_-v + \mu_w^2 - i\gamma_w\mu_w}, \quad \Delta_t = \frac{1}{-(x-\hat{y}_+)\hat{y}_-v + \mu_w^2 - i\gamma_w\mu_w}, \tag{100}$$

where

$$v = x_+x_-, \quad Y = \frac{1}{\hat{y}_+^2\hat{y}_-^2 + 4\gamma_w^2\mu_w^2}, \quad L = \ln \frac{M^2}{x\hat{y}_-\hat{s}}, \tag{101}$$

and  $\hat{y}_\pm, x$  are given in Eqs.(91, 92),  $\mu_w, \gamma_w$  after Eq.(71). With  $|A|^2$  at our disposal we can use Eq.(90) and Eq.(93), apply the convolution with QED structure functions, and derive the total cross section.

## 8 Numerical results and conclusions

In this section we present all relevant numerical results for the CC20 processes as computed by the FORTRAN program WTO version 2.0 [23]. The chosen setup is specified by the following list:

$$\sqrt{s} = 186 \text{ GeV}, \quad M_W = 80.39 \text{ GeV}, \quad M_Z = 91.1867 \text{ GeV} \quad (102)$$

Naive QCD is not introduced which implies, in particular, that the  $W$  width is included without QCD corrections. For our setup this results into  $\Gamma_W = 2.0459 \text{ GeV}$ . The QED radiation is included by means of the structure function approach (in the so-called  $\beta$ -scheme [20, 21, 22]). First we consider a cut on the scattering angle of the outgoing quarks with respect to the beam axis,  $10^\circ \leq \theta_q \leq 170^\circ$ . We also fix a lower cut on the invariant mass of the  $u\bar{d}$  system,  $M^2(u\bar{d}) \geq 0.01 s$ . According to the procedure described in Eq.(11) we introduce a separating angle  $\theta_c$  and compute the following cross sections:

$$\sigma_{<} \text{ or } |\text{CC20}_\gamma^<(m_e)|^2 \text{ for } \theta \leq \theta_c,$$

$$\sigma_{>} \text{ or } |\text{CC20}^>(0)|^2 \text{ for } \theta_c \leq \theta \leq \pi,$$

$$\sigma_{<\text{int}} \text{ or } 2 \left[ \text{CC20}_\gamma^<(0) \right]^\dagger \text{CC20}_R^<(0) + |\text{CC20}_R^<(0)|^2 \text{ for } \theta \leq \theta_c.$$

Our reference values will be  $\theta_c = 0.3^\circ, 0.4^\circ$  and  $0.5^\circ$ . We have verified that  $\sigma_{<\text{int}}$  is completely negligible for our choice of the separator  $\theta_c$  so that the total is safely given by the sum  $\sigma_{<} + \sigma_{>}$ . Indeed we find  $\sigma_{<\text{int}} = 6 \div 5 \div 3 \times 10^{-5} \text{ pb}$  for  $\theta_c = 0.5^\circ \div 0.4^\circ \div 0.3^\circ$ . Always for  $10^\circ \leq \theta_q \leq 170^\circ$  we find for  $\sigma_{<}/\sigma_{>}$  the results shown in Tab.(1).

$\theta_c [\text{Deg}]$	$\sigma_{<}$	$\sigma_{>}$	$\sigma_{<} + \sigma_{>}$
$0.3^\circ$	0.0527	0.6316(9)	0.6843(9)
$0.4^\circ$	0.0554	0.6289(7)	0.6843(7)
$0.5^\circ$	0.0575	0.6269(6)	0.6844(6)

Table 1: *Cross section in pb for the process  $e^+e^- \rightarrow e^-\bar{\nu}_e u\bar{d}$ , for  $10^\circ \leq \theta_q \leq 170^\circ$ ,  $q = u, d$  as a function of  $\theta_c$ .*

Tab.(1) clearly shows that there is a smooth matching of the two components,  $<(m_e)$  and  $>(0)$  at  $\theta = \theta_c$ . This result justifies the application of the WW-approximation in the narrow cone around the electron axis and we conclude by quoting the following result:

$$\sigma_{\text{CC20}}(186 \text{ GeV}, 10^\circ \leq \theta_q \leq 170^\circ, M^2(u\bar{d}) \geq 0.01 s) = 0.6843(4) \text{ pb}. \quad (103)$$

We have also varied the angular cut on the outgoing quarks, keeping  $M^2(u\bar{d}) \geq 0.01 s$  and  $\theta_c = 0.5^\circ$ . The latter is fully justified by the tiny dependence of the cross section on the separating angle  $\theta_c$ . The results are illustrated in Tab.(2).

$\theta_q$ [Deg]	$\sigma_{<}$	$\sigma_{>}$	$\sigma_{<} + \sigma_{>}$
$5^\circ$	0.0607	0.6404(6)	0.7011(6)
$6^\circ$	0.0601	0.6384(6)	0.6985(6)
$8^\circ$	0.0588	0.6333(6)	0.6921(6)
$10^\circ$	0.0575	0.6269(6)	0.6844(6)

Table 2: Cross section in pb for the process  $e^+e^- \rightarrow e^-\bar{\nu}_e u\bar{d}$ , for  $\theta_c = 0.5^\circ$  as a function of  $\theta_q$ .

Next, we consider the total CC20 cross section, without angular cuts on the outgoing quarks and with massless quarks. As explained in Sect. 7, where we have discussed QCD corrections, the resulting cross section depends on a factorization scale  $M$ . For  $M = 1 \text{ GeV}$  we find the results of Tab.(3). From

$\theta_c$ [Deg]	$\sigma_{<}$	$\sigma_{>}$	$\sigma_{<} + \sigma_{>}$
$0.3^\circ$	0.0850	0.6503(8)	0.7356(8)
$0.4^\circ$	0.0881	0.6473(7)	0.7354(7)
$0.5^\circ$	0.0905	0.6451(6)	0.7354(6)

Table 3: Cross section in pb for the process  $e^+e^- \rightarrow e^-\bar{\nu}_e u\bar{d}$ , for the factorization scale  $M = 1 \text{ GeV}$ , as a function of  $\theta_c$ .

Tab.(3) we derive the CC20 cross section in a fully extrapolated setup.

$$\sigma_{\text{CC20}}(186 \text{ GeV}, M = 1 \text{ GeV}) = 0.7354(4) \text{ pb.} \quad (104)$$

We have also investigated the dependence of the cross section on the factorization scale  $M$ . With  $\theta_c = 0.5^\circ$  and  $M^2(u\bar{d}) \geq 0.01s$  the results are presented in Tab.(4), showing a mild dependence on  $M$  of the total. Finally we have

$M$ [GeV]	$\sigma_{<}$	$\sigma_{>}$	$\sigma_{<} + \sigma_{>}$
0.1	0.0908	0.6451(6)	0.7359(6)
1	0.0905	0.6451(6)	0.7356(6)
10	0.0902	0.6451(6)	0.7353(6)
100	0.0900	0.6451(6)	0.7351(6)

Table 4: Cross section in pb for the process  $e^+e^- \rightarrow e^-\bar{\nu}_e u\bar{d}$ , for  $\theta_c = 0.5^\circ$  as a function of the factorization scale  $M$ .

analyzed single  $W$  production with  $\theta(e^-) < 0.5^\circ$  in WW-approximation. In Fig. 5 we have reported the  $M(u\bar{d})$  distribution for  $10^\circ \leq \theta_q \leq 170^\circ$ . In order to understand the role of the different components we have plotted the distribution with and without the multi-peripheral component. It follows that this component dominates at low invariant masses, while above  $M(u\bar{d}) \approx 70 \text{ GeV}$  it

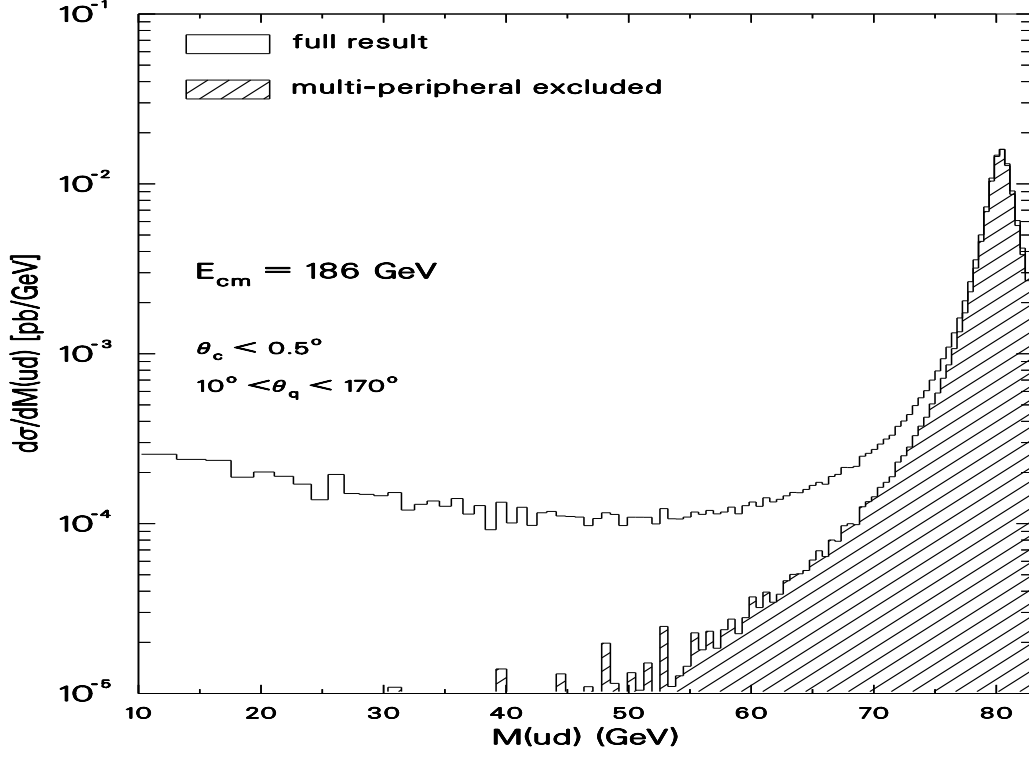


Figure 5: Invariant mass distribution for  $e^+e^- \rightarrow \bar{\nu}_e u \bar{d}$  showing the effect of the multi-peripheral component.

is practically without influence. A final comment is devoted to the validity of the WW-approximation. Note that we only use this approximation in a narrow cone around the electron axis, typically  $\theta \leq 0.5^\circ = 8.73 \text{ mrad}$ , and the complete calculation outside the cone. Corrections to Eq.(18) of  $\mathcal{O}(Q^2)$  have been discussed in [15] where it has been shown that, after integration, the cross section receives additional contributions proportional to powers of  $Q_c^2/(2Q \cdot p_+)$ . Since

$$\frac{Q_c^2}{2Q \cdot p_+} < \frac{s}{4s_0} \theta_c^2, \quad (105)$$

and, in our case,  $\sqrt{s} = 186 \text{ GeV}$ ,  $s_0 = 0.01 \text{ s}$  we find  $Q_c^2/(2Q \cdot p_+) < 1.9 \times 10^{-3}$ . Therefore non-factorizable corrections are formally negligible.

The mild dependence of the total cross section on the factorization scale  $M$  can be understood from Tab.(5). Here, for  $\theta \leq 0.5^\circ$ , we have reported: 1) the total cross section, i.e the sum of the single-resonant and of the multi-peripheral, non-resonant, components; 2) the multi-peripheral component alone. Therefore the non-resonant,  $M$ -dependent, terms are strongly suppressed and the  $M$ -dependence has little influence on the total, justifying our approximation of working at zeroth order in  $\alpha_s$ .

$M [\text{GeV}]$	$\sigma_{<}^{\text{TOT}}$	$\sigma_{<}^{\text{MP}}$
0.1	0.0908	0.0009
1	0.0905	0.0007
10	0.0902	0.0005
100	0.0900	0.0003

Table 5: *Total Cross section and Multi-Peripheral component in pb for  $e^+e^- \rightarrow e^-\bar{\nu}_e u \bar{d}$ , for  $\theta_c = 0.5^\circ$  as a function of the factorization scale  $M$ .*

The experimental Collaborations at LEP are now recording and analyzing a sizeable fraction of events with four fermions in the final state. Outgoing electrons represent a notorious problem because of the presence of  $t$ -channel photons interacting with  $W$  bosons or coupling to quark-antiquark pairs.

The collinear limit forbids a calculation where the massless limit for fermions is assumed from the beginning and, in turn, this may induce numerical instabilities in computing the total cross section, even for a fully massive MonteCarlo. It should be mentioned also that the majority of the MonteCarlos used in the analysis are built in the massless approximation and because of that the total cross section is not available.

We have suggested a simple but realistic solution based on the use of the Weizsäcker-Williams approximation, to be applied in a narrow cone around the beam axis. The large logarithms,  $\ln(m_e^2/s)$ , are correctly described by our numerical solution. Furthermore the improved WW-approximation that we are using is valid beyond the leading logarithmic approximation, as explained in Eq.(18), and correctly integrates also the  $m_e^2/Q^4$  terms present in the photon flux.

The correct treatment of the kinematics, accounting for the introduction of QED initial state radiation, is also emphasized. We have derived a version of the flux-function which describes quasi-real photons emitted by the electron after QED radiation. A second logarithmic enhancement in the cross section, arising from internal fermion propagators, is also described and a link is established with the familiar examples of  $\gamma p$  or  $\gamma\gamma$  scattering. Finally, several numerical results are shown, proving the goodness of the adopted solution.

## 9 Acknowledgements

I would like to express special thanks to Martin Grünewald for several important discussions on the experimental aspects of the problem, to Michelangelo Mangano for mentioning, long ago, the possible relevance of the Weizsäcker-Williams approximation in this context, to Alessandro Ballestrero for a discussion on the CC20 phase space, to Roberto Pittau for information on the status of EXCALIBUR.

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